IKKINCHI TARTIBLI HOSILASI KVADRATI BILAN INTEGRALLANUVCHI FUNKSIYALAR FAZOSIDA KVADRATUR FORMULANING EKSTREMAL FUNKSIYASI

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EXTREME FUNCTION OF THE QUADRATURE FORMULA IN THE SPACE OF FUNCTIONS INTEGRATED WITH A SECOND-ORDER DERIVATIVE.

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Annotation: In this paper, let's consider constructing an optimal quadrature formula in the mmm-space i.e. in the space of functions whose generalized derivative of order 2 is summed by its Square in the cross section $[0,1]$. Furthermore, using Rees's theorem on the general appearance of a linear continuous functional, finding the error-functional extreme function of a quadrature formula is brought to solve the boundary problem for Ordinary Differential Equations. By solving this boundary problem, a representation of the extreme function of the error functional of the quadrature formula is obtained.
Keywords: Quadrature formula, extreme Function, Error functional.

Annotatsiya: Ushbu ishda, $W_2^{(2)}(0,1)$ -fazosida ya'ni 2-tartibli umumlashgan hosilasi $[0,1]$kesmada kvadrati bilan jamlanuvchi funksiyalar fazosida optimal kvadratur formulani qurishni ko'rib chiqamiz. Bundan tashqari, chiziqli uzluksiz funksionalning

umumiy ko‘rinishi to‘g‘risidagi Riss teoremasidan foydalanib, kvadratur formulaning xatolik funksionali ekstremal funksiyasini topish oddiy differensial tenglamalar uchun chegaraviy masala yechishga keltiriladi. Bu chegaraviy masalani yechish orqali kvadratur formulaning xatolik funksionalining ekstremal funksiyasining ko‘rinishi olinadi.

Kalit so‘zlar: Kvadratur formula, ekstremal funksiya, xatolik funksionali.

Kirish. Masalani qo‘yilishi. Odatda amaliy masalalarni aniq integralni hisoblashga keltirilganda, ishning eng qiyin qismi allaqachon ortda qolganligini aytishimiz mumkin. Bundan tashqari, agar integral ostidagi funksiya $f(x)$ ga teng bo‘lsa, uning boshlang‘ichini $F(x)$funksiya elementari sifatida ifodalash mumkin bo‘lsa, unda aniq integralning qiymatini ushbu $\int_a^b f(x)dx = F(b) - F(a)$ formuladan foydalanib osongina olish mumkin. Amalda, bu ba’zan sezilarli qiyinchiliklarga olib kelishi mumkin, chunki integral ostidagi funksiyaning boshlang‘ichi $F(x)$ juda murakkab ko‘rinishga ega bo‘lishi mumkin.

Bu usul mutlaqo yaroqsiz bo‘lib chiqishi mumkin, agar (bu tez-tez sodir bo‘ladi) funksiyasi $F(x)$ elementar funksiyalar orqali ifodalanmaydi.

Alohida maxsus funksiyalar uchun ularning qiymatlari jadvalga kiritilgan, boshqa hollarda, aniq integral integralni u yoki bu turdagi qatorlarga yoyish orqali hisoblanishi mumkin.

Ba’zan ba’zi aniq integrallarni kompleks o‘zgaruvchining funksiyalari nazariyasidan foydalanib hisoblash mumkin.

Agar ushbu usullar mos bo‘lmasa, u holda aniq integralning qiymati raqamli integratsiya formulalari deb ataladigan formulalar yordamida raqamli tarzda hisoblanishi mumkin, ya’ni kvadratura formulalari.

Kvadratura usuli ko‘p hollarda boshqa usullarga nisbatan kamroq hisoblash kuchini talab qiladi. Ayrim integrallarning qiymatlarini eng yuqori aniqlikda va arzon narxlarda taxminiy hisoblash hisoblash matematikasining dolzarb muammolaridan biridir.

Qaralayotgan funksiyalar sinfida kvadratur formulaning maksimal xatoligi kvadratur formulalar nazaryasining muhim masalasi hisoblanadi.

Biz quyidagi ko‘rinishda kvadratur formulani qaraymiz [1-2]

$$\int_0^1 \varphi(x)dx \cong \sum_{\beta=0}^{N} k[\beta]\varphi(h\beta) \qquad (1)$$

bu yerda $k[\beta]$- (1) kvadratur formulaning hozircha noma'lum koeffitsiyentlari, $h=1/N$, $N-$ natural son, integral ostidagi $\varphi(x)$ funksiya $W_2^{(2)}(0,1)$ fazoga tegishli. Bunda $W_2^{(2)}(0,1)$ fazo bu birinchi tartibli hosilasi absolyut uzluksiz va ikkinchi tartibli hosilasi $L_2(0,1)$ ga tegishli barcha funksiyalar sinfi. Ushbu $W_2^{(2)}(0,1)$ fazo

$$\langle \varphi,\psi\rangle_W=\int_0^1\left(\frac{d^2\varphi(x)}{dx^2}\cdot\frac{d^2\psi(x)}{dx^2}+\frac{d\varphi(x)}{dx}\cdot\frac{d\psi(x)}{dx}\right)dx. \tag{2}$$

skalyar ko'paytmaga nisbatan Gilbert fazosi bo'ladi. Bu yerda (2)- skalyar ko'paytma yordamida

$$\|\varphi\|=\left\{\int_0^1\left(\left(\frac{d^2\varphi(x)}{dx^2}\right)^2+\left(\frac{d\varphi(x)}{dx}\right)^2\right)dx\right\}^{\frac{1}{2}} \tag{3}$$

aniqlanadi.

(1)-kvadratur formulaning xatoligi deb

$$\int_0^1\varphi(x)dx-\sum_{\beta=0}^{N}k[\beta]\varphi(h\beta) \tag{4}$$

ayirmaga aytiladi va bu ayirmaga $W_2^{(2)}(0,1)$ fazosida aniqlangan quydagi xatolik funksionali mos keladi

$$\ell(x)=i_{[0,1]}(x)-\sum_{\beta=0}^{N}k[\beta]\delta(x-h\beta)\ , \tag{5}$$

bu yerda $i_{[0,1]}(x)$-[0,1] kesmaning xarakteristik funksiyasi, $\delta(x)$-Dirakning delta-funksiyasi.

Odatda, $\ell(x)$ funksionalning $\varphi(x)$ funksiyadagi qiymati quyidagicha aniqlanadi

$$(\ell,\varphi)=\int_{-\infty}^{\infty}\ell(x)\varphi(x)dx\ . \tag{6}$$

(6) tenglikka asosan, (5) formulani etiborga olib, (4) ayirma haqiqatdan ham, $\ell(x)$ xatolik funksionalining $\varphi(x)$ dagi qiymati ekanligiga ishonch hosil qilamiz, ya'ni

$$(\ell,\varphi)=\int_{-\infty}^{\infty}\ell(x)\varphi(x)dx=\int_{-\infty}^{\infty}\left(i_{[0,1]}(x)-\sum_{\beta=0}^{N}k[\beta]\delta(x-h\beta)\right)\cdot\varphi(x)dx=$$

$$= \int_{-\infty}^{\infty} i_{[0,1]}(x)\varphi(x)dx - \sum_{\beta=0}^{N} k[\beta]\delta(x-h\beta)\cdot\varphi(x)\,dx = \int_{0}^{1}\varphi(x)dx - \sum_{\beta=0}^{N} k[\beta]\varphi(h\beta).$$

Demak,

$$(\ell,\varphi) = \int_{0}^{1}\varphi(x)dx - \sum_{\beta=0}^{N} k[\beta]\varphi(h\beta) \tag{7}$$

kvadratur formulaning (7) xatoligi $W_2^{(2)*}(0,1)$ fazosida chiziqli funksionalni aniqlaydi, bu yerda $W_2^{(2)*}$ fazo $W_2^{(2)}$ fazoga qo‘shma fazo. U holda, Koshi-Shvars tengsizligidan xatolikning absolyut qiymati yuqoridan quyidagicha baholanadi:

$$\left|(\ell,\varphi)\right| \le \left\|\varphi \mid W_2^{(2)}(0,1)\right\| \cdot \left\|\ell \mid W_2^{(2)*}(0,1)\right\|$$

Bu tengsizlikdan biz (1) kvadratur formulaning (7) xatoligi $\ell(x)$ xatolik funksionalining

$$\left\|\ell \mid W_2^{(2)*}(0,1)\right\| = \sup_{\left\|\varphi | W_2^{(2)}(0,1)\right\|=1} \left|(\ell,\varphi)\right| = \sup_{\varphi,\|\varphi\|\ne 0} \frac{\left|(\ell,\varphi)\right|}{\left\|\varphi \mid W_2^{(2)}(0,1)\right\|}$$

normasi orqali baholanishini xulosa qilamiz.

Yuqoridagi tengsizlikdan ko‘rinadiki, (4) xatolikning absolyut qiymatini yuqoridan baholash uchun, (5) xatolik funksionalining normasini topish talab etiladi. Buning uchun esa mos ekstremal funksiyani aniqlash kerak. Quyida aynan shu ekstremal funksiyani aniqlash masalasini qaraymiz.

Ushbu masalani yechish uchun, exstremal funksiya ta’rifidan foydalanamiz. Ma’lumki, Koshi-Shvarts tengsizligini tenglikka aylantiruvchi $U_\ell(x)$ funksiyaga ekstremal funksiya deyiladi, ya’ni

$$(\ell,U_\ell) = \left\|\ell \left| W_2^{(2)*}(0,1)\right.\right\| \cdot \left\|U_\ell \left| W_2^{(2)}(0,1)\right.\right\|. \tag{8}$$

$W_2^{(m)}(0,1)$ Gilbert fazosida chiziqli uzluksiz funksionalning umumiy ko‘rinishi haqidagi Riss teoremasidan foydalanib quyidagini yozamiz.

$$\left\|\ell \left| W_2^{(2)*}(0,1)\right.\right\| = \left\|U_\ell \left| W_2^{(2)}(0,1)\right.\right\|. \tag{9}$$

Shuningdek, (8) va (9) dan quyidagi xulosaga ega bo‘lamiz:

$$(\ell,U_\ell) = \left\|\ell \left| W_2^{(2)*}(0,1)\right.\right\|^2. \tag{10}$$

Boshqa tomondan, xuddi shu teorema bo‘yicha, $W_2^{(2)}(0,1)$ fazoning har qanday $\varphi(x)$ elementi uchun ushbu tenglikni olamiz

$$(\ell,\varphi)=\langle U_\ell,\varphi\rangle,$$

bu yerda

$$<U_\ell,\varphi>=\int_0^1\left(\frac{d^2\varphi(x)}{dx^2}\cdot\frac{d^2U_\ell(x)}{dx^2}+\frac{d\varphi(x)}{dx}\cdot\frac{dU_\ell(x)}{dx}\right)dx\quad.\qquad (11)$$

Aytaylik, $\varphi(x)$ funksiya $W_2^{(2)}(0,1)$ fazoga tegishli cheksiz differensiallanuvchi funksiya bo‘lsin, ya’ni.

$$\varphi(x)\in\overset{\circ}{C}{}^{(\infty)}(0,1).$$

(11) tenglikning o‘ng tomonini bo‘laklab integallab quyidagini hosil qilamiz

$$U_\ell^{(4)}(x)-U_\ell^{(2)}(x)=\ell(x).\qquad (12)$$

$$\left(U_\ell^{(3)}(x)-U_\ell^{(1)}(x)\right)\Big|_{x=0}^{x=1}=0,\qquad (13)$$

$$U_\ell^{(2)}(x)\Big|_{x=0}^{x=1}=0.\qquad (14)$$

Teorema 1. *(12)-(14) chegaraviy masalaning yechimi (1) kvadratur formulaning (5) xatolik funksionali ekstremal funksiyasi bo‘lib, quyidagi ko‘rinishga ega.*

$$U_\ell(x)=\ell(x)*\mu_2(x)+P_0,$$

bu yerda

$$\mu_2(x)=\frac{signx}{2}\left(\frac{e^x-e^{-x}}{2}-x\right)\qquad (15)$$

ushbu $\frac{d^4}{dx^4}-\frac{d^2}{dx^2}$ *differensial operatorning fundamental yechimi,*

$$signx=\begin{cases}1, & x>0,\\ 0, & x=0,\\ -1, & x<0,\end{cases}$$

P_0 *-ixtiyoriy o‘zgarmas son.*

Xulosa. Shunday qilib, biz ushbu ishda $W_2^{(2)}(0,1)$ fazoda optimal kvadratura formulasini qurishni ko‘rib chiqdik. Bundan tashqari, kvadratura formulalarining ekstremal funksiyasini topdik keying ishlarimizda xatolik funksionalining normasini ko‘rinishini topishni amalga oshiramiz. .

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