ODDIY DIFFERENSIAL TENGLAMALARNI MAPLE MATEMATIK PAKETI YORDAMIDA YECHISH USULLARI

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Differensial tenglamalarning har xil sinfidagi masalalarini yechish uchun, aynan formula bo‘yicha hisoblash, tenglamalar sistemasini yechish, belgili va sonli differensial tenglamalarni yechish amaliy jihatdan juda muhim masala hisoblanadi.

Ushbu maqolada Maple matematik paketidan foydalanib, defferensial tenglamalarni yechish keltirilgan. Maple paketi orqali defferensial tenglamalarni yechish jarayoni qoidaga mos holda ta’lim berish uchun qiziqarli misollar yordamida tasvirlangan. Maple paketining har bir turdagi masalani yechishga qo‘llanilishi ketma-ket tarzda keltirilgan, ya’ni defferensial tenglamalarni yechishda misollarga quyidagicha tavsif berilgan: hisoblash formulasi, analitik va sonli yechimi, shuningdek, defferensial tenglamalarning grafigi keltirilgan.

n - chi tartibli o‘zgarmas koeffisiyentli birjinsli differensial tenglamalarni qarab chiqamiz [7].

$$a_0 y^{(n)} + a_1 y^{(n-1)} + \dots + a_n y = 0 \qquad (1)$$

tenglamaga *n* - chi tartibli o‘zgarmas koeffisiyentli birjinsli differensial tenglama deyiladi. Bu yerda $a_0, a_1, \dots, a_n$ o‘zgarmas sonlar.

Tenglamaning xususiy yechimi $y = e^{\lambda x}$ ko‘rinishda bo‘lib, u

$$a_0 \lambda^n + a_1 \lambda^{n-1} + \dots + a_n = 0 \qquad (2)$$

λ - xarakteristik tenglamaning ildizi bo‘lishi kerak. Yechim ko‘rinishi (2) xarakteristik tenglama ildizlariga bog‘liq:

a) (2) tenglamaning barcha ildizlari haqiqiy va har xil.

Bu holda $y_1 = e^{\lambda_1 x}, y_2 = e^{\lambda_2 x}, \dots, y_n = e^{\lambda_1 x}$ yechimlar tenglamaning fundamental yechimlar sistemasini tashqil etadi, chunki ular yordamida tuzilgan Vronskiy determinanti noldan farqli.

1-misol. $y'' - 7y' + 12y = 0$.

Xarakteristik tenglamani tuzamiz

$$\lambda^2 - 7\lambda + 12 = 0.$$

λ=3, λ=4 bu tenglamaning ildizlaridir. Demak, $y_1 = e^{3x}, y_2 = e^{4x}$ tenglamaning hususiy yechimlari va $y = c_1 e^{3x} + c_2 e^{4x}$ berilgan tenglamaning umumiy yechimi bo‘ladi.

1-misolni Maple paketi orqali yechamiz [5]:

> **restart;**

> **d:=diff(y(x),x$2)-7*diff(y(x),x)+12*y(x);**

$$d := \left(\frac{d^2}{dx^2}\,y(x)\right) - 7\left(\frac{d}{dx}\,y(x)\right) + 12\,y(x)$$

> **dsolve({d},y(x));**

$$\{y(x) = \_C1\,e^{(3\,x)} + \_C2\,e^{(4\,x)}\}$$

b) (2) tenglamaning ildizlari orasida kompleks yechim mavjud.

Xarakteristik tenglama haqiqiy koyeffisiyentli bo‘lganligi sababli ildizga qo‘shma bo‘lgan son ham ildiz bo‘ladi. Bu ildizlar $\lambda_1 = \alpha + \beta i,\ \lambda_2 = \alpha - \beta i,$ bo‘lsin. Bu ildizlarga (1) tenglamaning $y_1 = e^{\lambda x} Cos\beta x, y_2 = e^{\lambda x} Sin\beta x$ ko‘rinishdagi ikkita yechim mos keladi.

2-misol. $y'' + 4y' + 13y = 0$.

Xarakteristik tenglama $\lambda^2 + 4\lambda = 13 = 0$. U $\lambda_{1,2} = -2 \pm 3i$ ildizlarga ega, demak, $y_1 = e^{-2x} Cos3x, y_2 = e^{-2x} Sin3x$ berilgan tenglamaning xususiy yechimlari bo‘lib, ular chiziqli bog‘lanmagan va

$$y = c_1 e^{-2x} Cos3x + c_2 e^{-2x} Sin3x$$

tenglamaning umumiy yechimi bo‘ladi.

2-misolni Maple paketi orqali yechamiz:

> **restart;**

> **d:=diff(y(x),x$2)+4*diff(y(x),x)+13*y(x);**

$$d := \left(\frac{d^2}{dx^2}\,y(x)\right) + 4\left(\frac{d}{dx}\,y(x)\right) + 13\,y(x)$$

> **dsolve({d},y(x));**

$$\{y(x) = \_C1\,e^{(-2\,x)}\sin(3\,x) + \_C2\,e^{(-2\,x)}\cos(3\,x)\}$$

c) Xarakteristik tenglamaning ildizlari orasida karrali ildiz mavjud.

Masalan, λ_1 tenglamaning r karrali ildizi bo‘lsin, bu holda (1) tenglama r ta

$$y_1 = e^{\lambda_1 x}, y_2 = xe^{\lambda_1 x}, \ldots, y_r = x^{r-1} e^{\lambda_1 x} \quad (3)$$

ko'rinishdagi hususiy yechimga ega bo'ladi. Bu yechimlarni chiziqli bog'lanmaganligini bevosita Gram determinantidan foydalanmasdan aniqlash mumkin.

$$(c_1 + c_2 x + \ldots + c_r x^{r-1}) e^{\lambda_1 x} = 0 \qquad (4)$$

tenglik barcha x lar uchun o'rinli bo'lsin, u holda

$$c_1 + c_2 x + \ldots + c_r x^{r-1}$$

ko'phad aynan nolga teng bo'ladi, bu esa ko'phadning barcha koeffisiyentlari nol bo'lgandagina bajarilishi mumkin. Demak, (4) tenglik faqat $c_1 = c_2 = \ldots = c_r = 0$ bo'lganda bajariladi va bundan (3) chiziqli bog'lanmagan funksiyalar sistemasini tashqil etadi.

3-misol. $y''' + 3y'' + 3y' + y = 0.$

Xarakteristik tenglama $\lambda^3 + 3\lambda^2 + 3\lambda + 1 = 0$. U $\lambda_{1,2,3} = 1$ uch karrali ildiz bo'lganligi sababli tenglamaning xususiy yechimlari e^x, xe^x, $x^2 e^x$ bo'ladi.

Shunday qilib, tenglamaning umumiy yechimi quyidagi ko'rinishga ega

$$y = (c_1 + c_2 x + c_3 x^2) e^x.$$

3-misolni Maple paketi orqali yechamiz:

```
> restart;
> d:=diff(y(x),x$3)+3*diff(y(x),x$2)+3*diff(y(x),x)+y(x);
```

$$d := \left(\frac{d^3}{dx^3} y(x)\right) + 3\left(\frac{d^2}{dx^2} y(x)\right) + 3\left(\frac{d}{dx} y(x)\right) + y(x)$$

```
> dsolve({d},y(x));
```

$$\{ y(x) = \_C1\, e^{(-x)} + \_C2\, e^{(-x)} x + \_C3\, e^{(-x)} x^2 \}$$

O'zgarmas koeffisiyentli bir jinsli chiziqli differensial tenglamalar uchun qo'yilgan Koshi masalasi yechish uchun

$$a_0 y^{(n)} + a_1 y^{(n-1)} + \ldots + a_n y = f(x) \qquad (5)$$

tenglamani qaraymiz. Bu yerda $a_0, a_1, \ldots, a_n$ - o'zgarmas sonlar, $f(x)$ $x \in [a,b]$ da aniqlangan va uzluksiz funksiya.

$$a_0 y^{(n)} + a_1 y^{(n-1)} + \ldots + a_n y = 0 \qquad (6)$$

tenglamaga n - chi tartibli o'zgarmas koeffisiyentli chiziqli birjinsli differensial tenglama deyiladi. Bu yerda $a_0, a_1, \ldots, a_n$ o'zgarmas sonlar.

O‘zgarmas koeffisiyentli birjinsli chiziqli differensial tenglamalar uchun Koshi masalasi $y(x_0)=y_0,\ y'(x_0)=y_0',\ \dots,\ y^{(n-1)}(x_0)=y_0^{(n-1)}$ shartlarni qanoatlantiruvchi yechimni topishdan iboratdir [1,2].

5-Misol. $y''-3y'+2y=0$ differensial tenglamaning $y(0)=-1,\ y'(0)=3$ boshlang‘ich shartni qanoatlantiruvchi xususiy yechimini toping.

Yechish: Xarakteristik tenglamani tuzamiz $r^2-3r+2=0$, uning ildizlari $r_1=1, r_2=2$ haqiqiy har xil sonlar, shuning uchun tenglamaning umumiy yechimi

$$y=c_1e^x+2c_2e^{2x} \qquad (*)$$

ko‘rinishda bo‘ladi. c_1 va c_2 ning qiymatlarini topish uchun $x=0$ da, $y=-1$ boshlang‘ich shartlardan foydalanib quyidagi tenglamani tuzamiz

$$c_1+c_2=-1$$

(*) differensiallab, topamiz

$$y'=c_1e^x+2c_2e^{2x},$$

bundan $y'(0)=3$ dan foydalanib, $c_1+2c_2=3$ ni hosil qilamiz. Shunday qilib c_1 va c_2 larni topish uchun quyidagi sistemani topdik.

$$\begin{cases} c_1+c_2=-1 \\ c_1+2c_2=3 \end{cases},$$

buni yechib $c_1=\text{-}5, c_2=4$ topamiz. Umumiy yechimga c_1 va c_2 larning qiymatini qo‘yib, berilgan tenglamaning xususiy yechimini topamiz:

$$y=\text{-}5e^{\text{-}x}+4e^{2x}.$$

5-misolni Maple paketi orqali yechamiz:

> **restart; Order:=6:**

> **d:=diff(y(x),x$2)-3*diff(y(x),x)+2*y(x);**

$$d:=\left(\frac{d^2}{dx^2}\mathrm{y}(x)\right)-3\left(\frac{d}{dx}\mathrm{y}(x)\right)+2\,\mathrm{y}(x)$$

Differensial tenglamaning umumiy yechimi quyidagicha bo‘ladi:

> **dsolve({d},y(x));**

$$\{\mathrm{y}(x)=\_C1\,\mathrm{e}^x+\_C2\,\mathrm{e}^{(2\,x)}\}$$

Endi boshlang‘ich shartlarni kiritamiz:

> **cond:=y(0)=-1, D(y)(0)=3;**

$$cond:=\mathrm{y}(0)=\text{-}1,\ \mathrm{D}(y)(0)=3$$

Tenglamaning boshlang‘ich shartlarni qanoatlantiruvchi xususiy yechimi quyidagiga teng:

```
> dsolve({d,cond},y(x));
```

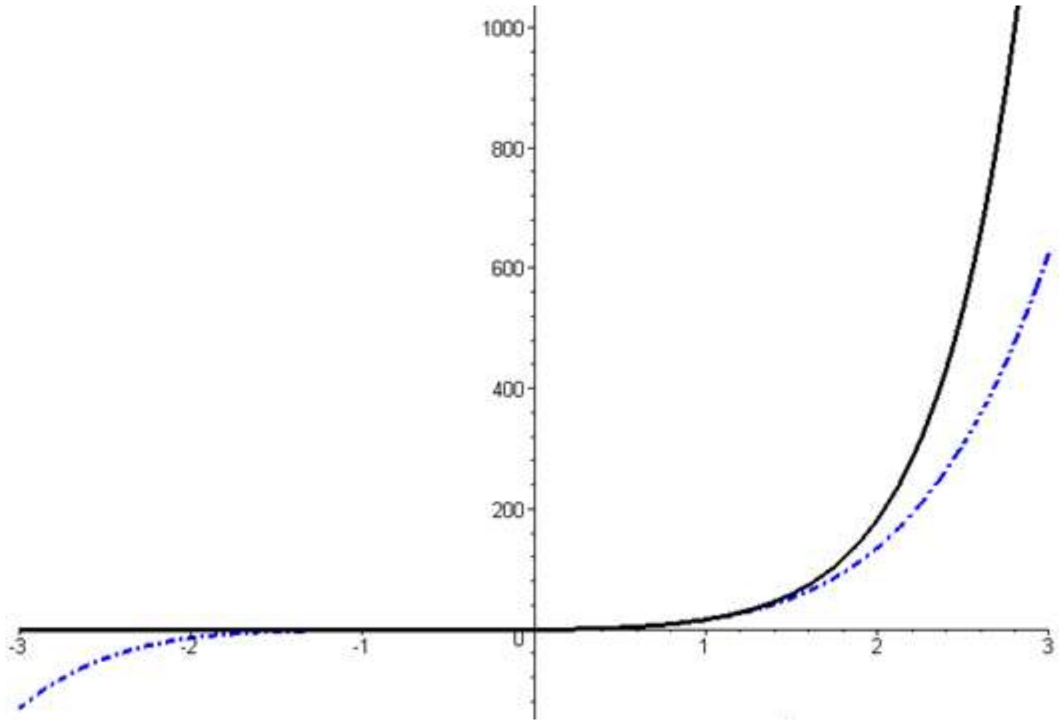
$$y(x) = -5\,e^{x} + 4\,e^{(2x)}$$

```
> y1:=rhs(%):
> dsolve({d,cond},y(x), series);
```

$$y(x) = -1 + 3x + \frac{11}{2}x^2 + \frac{9}{2}x^3 + \frac{59}{24}x^4 + \frac{41}{40}x^5 + O(x^6)$$

Topilgan yechimning grafigini quyidagicha bo‘ladi:

```
> convert(%,polynom): y2:=rhs(%):
> p1:=plot(y1,x=-3..3,thickness=3,color=black):
> p2:=plot(y2,x=-3..3, linestyle=4,thickness=3,color=blue):
> with(plots): display(p1,p2);
Warning, the name changecoords has been redefined
```



6-misol. $\begin{cases} y^{(IV)} - 5y'' + 10y' - 6y = 0, \\ y(0) = 1,\ y'(0) = 1, y''(0) = 1,\ y'''(0) = 0,\ y^{(4)}(0) = 1. \end{cases}$ o‘zgarmas koeffitsentli chiziqli bir jinsli differensial tenglamani va unga qo‘yilgan Koshi masalasini yeching. Yechimni maple paketi orqali topamiz [6]:

```
> restart; Order:=6:
> d:=diff(y(x),x$4)-5*diff(y(x),x$2)+10*diff(y(x),x)-6*y(x);
```

$$d := \left(\frac{d^4}{dx^4}y(x)\right) - 5\left(\frac{d^2}{dx^2}y(x)\right) + 10\left(\frac{d}{dx}y(x)\right) - 6\,y(x)$$

Differensial tenglamaning umumiy yechimi quyidagicha bo‘ladi:

```
> dsolve({d},y(x));
```

$$\{y(x) = \_C1\, e^{(-3x)} + \_C2\, e^{x} + \_C3\, e^{x}\sin(x) + \_C4\, e^{x}\cos(x)\}$$

Endi boshlang‘ich shartlarni kiritamiz:

```
> cond:=y(0)=1, D(y)(0)=1, (D@@2)(y)(0)=1, (D@@3)(y)(0)=0, (D@@4)(y)(0)=1;
```

$$cond := y(0)=1,\ D(y)(0)=1,\ (D^{(2)})(y)(0)=1,\ (D^{(3)})(y)(0)=0,\ (D^{(4)})(y)(0)=1$$

Tenglamaning boshlang‘ich shartlarni qanoatlantiruvchi xususiy yechimi quyidagiga teng:

```
> dsolve({d,cond},y(x));
```

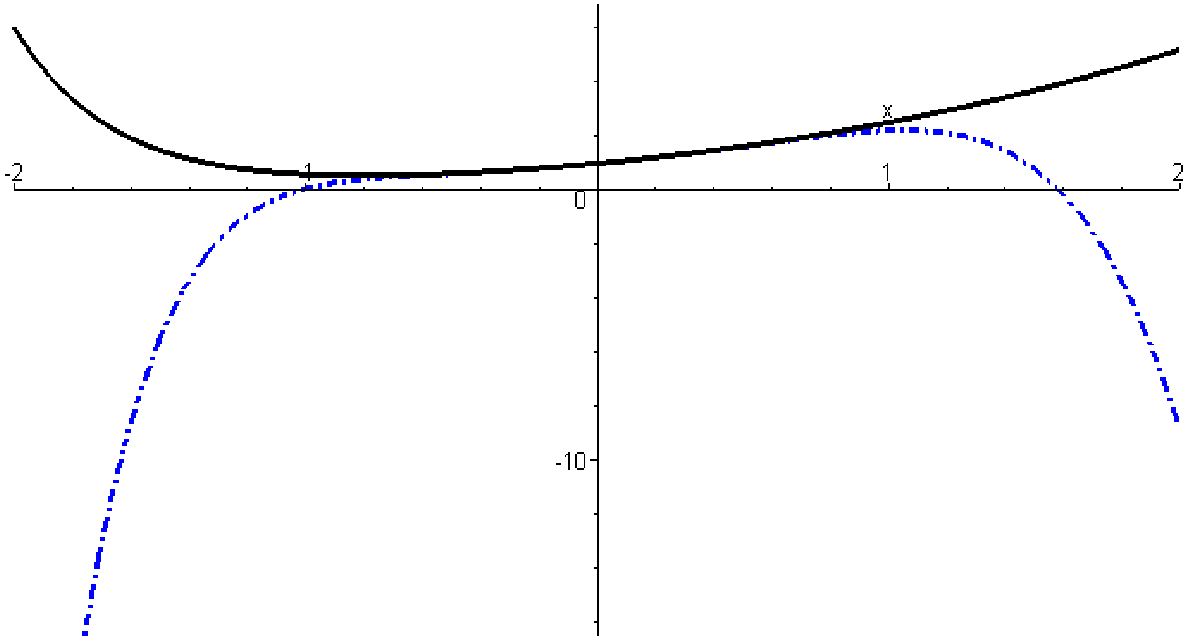
$$y(x) = \frac{1}{68}e^{(-3x)} + \frac{3}{4}e^{x} + \frac{1}{17}e^{x}\sin(x) + \frac{4}{17}e^{x}\cos(x)$$

```
> y1:=rhs(%):
> dsolve({d,cond},y(x), series);
```

$$y(x) = \frac{5}{6}x^3\left(1 + \frac{1}{4}x^2 - \frac{1}{12}x^3 + \frac{31}{840}x^4 - \frac{5}{336}x^5 + O(x^6)\right) - \frac{1}{8640}x^2\left(17280 + 7200\,x^2 - 2880\,x^3 + 1488\,x^4 - \frac{4800}{7}x^5 + O(x^6)\right) - \frac{1}{576}x\left(-576 - 480\,x^2 + 240\,x^3 - \frac{744}{5}x^4 + 80\,x^5 + O(x^6)\right) - \frac{1}{144}\left(-144 - 360\,x^2 + 240\,x^3 - 186\,x^4 + 120\,x^5 + O(x^6)\right)$$

Topilgan yechimning grafigini quyidagicha bo‘ladi:

```
> convert(%,polynom): y2:=rhs(%):
> p1:=plot(y1,x=-2..2,thickness=3,color=black):
> p2:=plot(y2,x=-2..2, linestyle=4,thickness=3,color=blue):
> with(plots): display(p1,p2);
```



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