AN UPPER BOUND OF REAL ZEROS OF A RANDOM POLYNOMIAL

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Abstract— A new upper bound for the number of real zeros of a random algebraic polynomial with real coefficients is obtained. It is supposed that the coefficients are in dependent random variables identically distributed with expectation value zero, the variance and the third absolute moment being finite and non-zero.
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1. INTRODUCTION

Let Nn be the number of real roots of the algebraic equation. $\wedge\ \wedge\ _v x^v = 0\ v=0$
where Zo, ZI Z2 Zn are independent random variables assuming real values only.
Littlewood and Offord [2] have estimated that $Nn<25(\log n)^2$
Outside an exceptional set E where p(E) <(12 log n)/n, n being sufficiently large.
They supposed that Z coefficients are either normally distributed, or uniformly distributed in (-1, +1), or assume only the values +1 and -1 with equal probabilities.
Erdos and Offord [1] proved that N_{t5}***= —logn + Ol(logn)^3.logni***
Π
Outside an exceptional set of measure not exceeding a proportion O '(log log n) ^{3}***1 of the total number of equations.***
Samal and Mishra [5] have considered the case that the Z-coefficients have a common characteristic function
$\exp^{(-C|t|^{a})}$
where C is a positive constant and 1<a<2. They have shown that

Nn < Mlog^{n})}2

outside an exceptional set of measure O(1/n), ц being a positive constant.

Logan and Shepp [3] have studied the above case when 0<a<2 and have obtained that

$^{E(}$Nn$^{)}$~C« logn

as n tends to infinity. Using this result, one gets that N$_{t2}$< {g(log n)}2

outside an exceptional set of measure O(1/n), as n tends to infinity. Samal [4] has considered the general case that the Z-coefficients have identical distributions with exception value zero, the variance and the third absolute moment being finite and non-zero. He $^{(}$plogn + $^{2)l}$og$^{(}$pn$^{3)}$, log-in^{3})

has shown (pp. 436-7) that N < ---------------1 -----------

^{n} log 2 log2(1 - p)

where p is any number greater than 1/log2 and 0<p<1/2. As n tends to infinity, this bound is asymptotic to

3p(log$^{n)2}$

log 2

3p(logn)^{2}

So that Samal has proved that, outside an exceptional set of small probability N$_{t5}$

<---

log2

Our aim is to improve the upper bound obtained by Samal [4]. We show that

Nn< 7-pT^{f5} + 2 Vogn $^{-}$ loglogn$^{)lo}$g^{n}

log2 ^2 в J

where p>1/log2 and в>4.

2. THEOREM ***n***

Let ^^$_{v}$x^{v} = 0 be a polynomial of degree n whose coefficients Z0, Z1, Z2, Zn are independent real random variables

v=0

identically distributed with exception value zero, the variance and the third absolute moment being finite and non-zero. Then there exists an integer n0 large enough such that, for n>n0, the number of real roots of most of the equations f(x)=0 does not exceed

y-pr ^{f} 5 $^{+}$ 2 Ilogn — loglogn)logn log2 ^ 2 в J

where p>1/log2 and в>4. The measure of the exceptional set is

$0\{(\log\log n)^2(\log n)^{-1/2}\}$.

Proof:-***Choose p a fixed real number greater than 1/log2 and let k be given by k=[plogM]***

Where M=en/1ogn, в>0 and [x] denotes the greatest integer not exceeding x. We consider k+1 circles C_m ***(m=1,2, k, plogM), Cm having its center at xm=1-(1/2m) and a radius*** $r_m = (1 - x_m)/2 = 1/2^{m+1}$***. In addition, we also consider the circle*** C_0 ***with its center at*** $x_0=1$ ***and radius*** $r_0=1/m$***. The circles*** C_0***, C1, ... Ck,*** C_{plogM} ***cover the closed segment [1/2,1]. Indeed, first of all, their centers are points belonging to that segment. Furthermore, C1,*** C_2 ***and C3 .... Ck and*** Cp_{logm} ***intersect each other two by two. Finally, Cplogm and C0 also intersect each other, since***

11 1 1

r +1 r.—|xr.— X) = —.-----; ------- 1 - ;

p logM 0 0 p logM plogm plogm

1 11 11 1

. ---; = --. — > 0 M 2 2plogm M 2 Mplog2

taking into account that p>1log2.

Next, for every m, let r_m ***represent the circle concentric with Cm and with radius*** $2r_m.r_m$ ***where m=1, 2 .... k, ...plogM, covers the*** segment $[1 - (1/2m^1),1]$ ***whereas r0 covers the segment*** $[1-(1/2M),1+(2/M)]$ ***and so all r-circles are interior to*** $M=^{1+}m$

Now, we appeal to the well-known result that the number of zeros of a regular function O(z) in a circle with center at z_0 ***and radius***

M

a t a 1™Φ(^1

r does not exceed log---——

log — r

where M is an upper bound of |ϕ| in a concentric circle of radius R. This is a direct consequence of Jensen's theorem. In the

present case, putting $Z_{(}$, =x_m, R = 2$r_m$and r =r_m. We find that the number of zeros of any polynomial f(x) in Cm is at most.

maxl$^{f(z)}$l ^{Z}^m _____

f (x) log------- ^{m}

log 2

But, since rm is interior to the circle |z| = 1 + (2 / M), we also have:

maxl$^{f(z)}$l max |$^{f(z)}$|

Zer$_m$ Z<1+(2/M)

f (x) f (x)

log ~~; m~~ < log-------------' (1)

log2 log2

At this point, we need an upper bound for the Z-coefficients. For this, we make use of Chebyshev's inequality concerning the

probability that a random variable, taken in absolute value, exceeds a given positive number. In this way, there comes:

Pr(U £ (n + 1)1/2 logn)< -^------ - and hence,

(n + 1)(logn)^2

2

Pr(^ I < (n +1)1/2 logn,0 < v < n)> 1 --- - where o^2 is the variance of the Z-coefficients. Thus, outside a set with a

(logn)^2

G 2 measure which is at most , we have :

(login)^2