# KOSHI-SHVARTS VA GYOLDER TENGSIZLIKLARI YORDAMIDA BA’ZI OLIMPIADA MASALALARINI YECHISH USULLARI

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***Annotatsiya.*** Ushbu maqolada Koshi-Shvarts va Gyolder tengsizliklari yordamida ba’zi olimpiada masalalarini yechish usullari keltirilgan. Koshi-Shvarts va Gyolder tengsizliklari matematikada muhim ahamiyatga ega bo‘lib, ular ko‘plab olimpiada masalalarida qo‘llaniladi. Ushbu tengsizliklarning asosiy afzalliklari ularning qamrovli qo‘llanilishi va parametr sifatida o‘zgartirish kiritish mumkinligi bilan bog‘liq.

***Kalit so‘zlar:*** Koshi-Shvarts tengsizligi, Gyolder tengsizligi, proportsional sonlar, olimpiada masalalari, musbat haqiqiy sonlar.

Misollar ko‘rishdan oldin Koshi-Shvarts va Gyolder tengsizliklari bilan qisqacha tanishib chiqamiz.

**Koshi-Shvarts tengsizligi.** Har qanday $a_1, a_2, ..., a_n$ va $b_1, b_2, ..., b_n$ haqiqiy sonlar uchun quyidagi tengsizlik bajariladi:

$$\left(a_1^2 + a_2^2 + ... + a_n^2\right)\left(b_1^2 + b_2^2 + ... + b_n^2\right) \text{i} \left(a_1 b_1 + a_2 b_2 + ... + a_n b_n\right)^2.$$

Bu yerda $a_1, a_2, ..., a_n$ va $b_1, b_2, ..., b_n$ sonlari proportsional bo‘lgan holdagina tenglik belgisi bajariladi. Ya’ni $\frac{a_1}{b_1} = \frac{a_2}{b_2} = ... = \frac{a_n}{b_n}$.

Gyolder tengsizligi bu Koshi-Shvarts tengsizligining umumiylashgan holi bo‘lib, u quyidagicha.

**Gyolder tengsizligi.** Aytaylik $a_{i_j}$, $1 \le i \le m$, $1 \le j \le n$ sonlar musbat haqiqiy sonlar bo‘lsin. U holda bu sonlar uchun quyidagi tengsizlik bajariladi

$$\prod_{i=1}^{m}\left(\sum_{j=1}^{n} a_{i_j}\right) \geq \left(\sum_{j=1}^{n} \sqrt[m]{\prod_{i=1}^{m} a_{i_j}}\right)^m .$$

Bu tengsizlikni tushunish birmuncha qiyin ko‘ringanligi sababli soddalik maqsadida $a,b,c,p,q,r,x,y,z$ musbat haqiqiy sonlar uchun ohirgi tengsizlikning quyidagi xususiy holini qaraymiz:

$$\left(a^3+b^3+c^3\right)\left(p^3+q^3+r^3\right)\left(x^3+y^3+z^3\right)\geq\left(aqx+bqy+crz\right)^3.$$

***1-misol.*** $a$, $b$ va $c$ haqiqiy sonlar uchun

$$3\left(a^2+b^2+c^2\right)\geq(a+b+c)^2$$

tengsizlik o‘rinli bo‘lishini isbotlang.

***Isboti.*** Demak, Koshi-Shvarts tengsizligiga ko‘ra

$$\left(1^2+1^2+1^2\right)\left(a^2+b^2+c^2\right)\geq(1\cdot a+1\cdot b+1\cdot c)^2.$$

***2-misol.*** $x$, $y$, $z$ manfiy bo‘lmagan sonlar uchun quyidagi tengsizlik o‘rinli bo‘lishini isbotlang:

$$\sqrt{3x^2+xy}+\sqrt{3y^2+yz}+\sqrt{3z^2+zx}\leq 2(x+y+z).$$

***Isboti.*** Koshi-Shvarts tengsizligiga binoan

$$\sum_{cyc}\sqrt{x(3x+y)}\leq\sqrt{\left(\sum_{cyc}x\right)\left(\sum_{cyc}(3x+y)\right)}=\sqrt{4(x+y+z)^2}=2(x+y+z).$$

***3-misol.*** Musbat haqiqiy $a,b,c$ sonlar ushun quyidagi tengsizlik o‘rinli bo‘lishini isbotlang:

$$\frac{a}{2a+b}+\frac{b}{2b+c}+\frac{c}{2c+a}\leq 1.$$

***Isboti.*** Ma’lumki

$$\sum_{cyc}\frac{a}{2a+b}\leq 1$$

$$\Leftrightarrow\quad\sum_{cyc}\left(\frac{a}{2a+b}-\frac{1}{2}\right)\leq 1-\frac{3}{2}$$

$$\Leftrightarrow\quad -\frac{1}{2}\sum_{cyc}\frac{b}{2a+b}\leq-\frac{1}{2}$$

$$\Leftrightarrow\quad\sum_{cyc}\frac{b}{2a+b}\geq 1.$$

Demak, Koshi-Shvarts tengsizligini qo‘llash natijasida biz quyidagiga ega bo‘lamiz

$$\sum_{cyc}\frac{b}{2a+b}=\frac{b^2}{2ab+b^2}+\frac{c^2}{2bc+c^2}+\frac{a^2}{2ca+a^2}\geq\frac{(a+b+c)^2}{2(ab+bc+ca)+b^2+c^2+a^2}=1.$$

***4-misol.*** $x, y, z$ musbat haqiqiy sonlar uchun quyidagi

$$\frac{y^2+z^2}{x}+\frac{z^2+x^2}{y}+\frac{x^2+y^2}{z}\geq 2(x+y+z).$$

tengsizlik o‘rinli bo‘lishini isbotlang.

***Isboti.***

$$\frac{y^2+z^2}{x}+\frac{z^2+x^2}{y}+\frac{x^2+y^2}{z}=\left(\frac{y^2}{x}+\frac{z^2}{y}+\frac{x^2}{z}\right)+\left(\frac{z^2}{x}+\frac{x^2}{y}+\frac{y^2}{z}\right)$$

$$\geq\frac{(y+z+x)^2}{x+y+z}+\frac{(z+x+y)^2}{x+y+z}=2(x+y+z).$$

***5-misol.*** Barcha musbat haqiqiy $a$, $b$ va $c$ sonlar uchun

$$\frac{a}{\sqrt{a^2+8bc}}+\frac{b}{\sqrt{b^2+8ca}}+\frac{c}{\sqrt{c^2+8ab}}\geq 1$$

tengsizlik o‘rinli bo‘lishini isbotlang.

***Isboti.*** Gyolder tengsizligiga ko‘ra ushbu

$$\left(\sum_{cyc}\frac{a}{\sqrt{a^2+8bc}}\right)^2\left(\sum_{cyc}a\left(a^2+8bc\right)\right)\geq(a+b+c)^3$$

tengsizlik bajariladi. Shuning uchun $(a+b+c)^3\geq a^3+b^3+c^3+24abc$ tengsizlik bajarilishini ko‘rsatishimiz kifoya. Bizga ma’lum bu tengsizlik barcha musbat haqiqiy sonlar uchun bajariladi.

***6-misol.*** Aytaylik $p\geq 2$ haqiqiy son bo‘lsin. U holda barcha manfiy bo‘lmagan haqiqiy $a,b,c$ sonlar uchun

$$\sqrt[3]{\frac{a^3+pabc}{1+p}}+\sqrt[3]{\frac{b^3+pabc}{1+p}}+\sqrt[3]{\frac{c^3+pabc}{1+p}}\leq a+b+c$$

tengsizlik bajarilishini isbotlang.

***Isbot.*** Demak, Gyolder tengsizligi bo‘yicha

$$\left(\sum_{cyc}\sqrt[3]{\frac{a^3+pabc}{1+p}}\right)^3=\left(\sum_{cyc}\sqrt[3]{\frac{1}{1+p}\cdot a\cdot\left(a^2+pbc\right)}\right)^3\le\left(\sum_{cyc}\frac{1}{1+p}\right)\left(\sum_{cyc}a\right)\left(\sum_{cyc}a^2+pbc\right).$$

Bu yerda $a^2+b^2+c^2\ge ab+bc+ca$ bo‘lganligi uchun

$$\sum_{cyc}a^2+pbc\le(p+1)\sum_{cyc}\frac{a^2+2bc}{3}=\frac{p+1}{3}(a+b+c)^2$$

tengsizlikni olamiz. Demak, so‘ngi tengsizlikdan

$$\left(\sum_{cyc}\sqrt[3]{\frac{a^3+pabc}{1+p}}\right)^3\le\frac{3}{p+1}\cdot(a+b+c)\cdot\frac{p+1}{3}(a+b+c)^2=(a+b+c)^3$$

isbotlanishi talab etilgan tengsizlikni olamiz.

**ADABIYOTLAR**

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