TERMOELASTIKLIK NAZARIYASI MASALALARINING KUCHLANISHLARGA NISBATAN MODEL TENGLAMASI

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Annotatsiya. Termoelastiklik nazariyasi masalalarni odatda kuchlanishlarda yechishda Eri kuchlanish funksiyasi kiritib yechilgan lekin biz bu ishda hech qanday qo'shimcha funksiya kiritmasdan to'g'ridan tog'ri kuchlanishlarga nisbatan termoelastiklik nazariyasi masalasini fazoviy holatda qo'yilishi va yechish algoritmi ko'rsatib o'tilgan**.**

Kalit so'zlar. Beltrami-Mitchell tenglamalari, iteratsiya, kuchlanish, kuchlanish funksiyasi.

MODEL EQUATION OF THERMOELASTICITY THEORY PROBLEMS IN TERMS OF STRESS

Abstract. In the theory of thermoelasticity, problems are typically solved in terms of stress by introducing Airy's stress functions. However, in this work, the problem of thermoelasticity is formulated and solved directly in terms of stresses in a spatial setting, without introducing any additional functions. The algorithm for the solution is also presented.

Keywords: Beltrami-Mitchell equations, iteration, stress, stress function.

Ko'p hollarda konstruksiyalar va ularning elementlarining deformatsiyalanish jarayoni termomexanik kuchlar ta'sirida sodir bo'lib, bu jarayon qattiq jismlarda issiqlik ajralishi hamda haroratning oshishi bilan kechadi. Issiqlik tarqalish jarayonini tasvirlovchi matematik modelni birinchi marta Dyugamel–Neyman ishlarida [4-7] ko'rib chiqilgan bo'lib, unda to'liq deformatsiya elastik deformatsiya va termik kengayish deformatsiyasidan iborat deb hisoblangan. Deformatsiyalanadigan qattiq jismlarning termoelastiklik nazariyasi masalalari quyidagi ishlarda tadqiq etilgan [2-3]. Odatda, termoelastiklik masalalarini yechishda temperatura issiqlik o'tkazuvchanlik tenglamasi yechimi sifatida ma'lum deb olinadi va haroratga bog'liq bo'ladi. Bunday masalalar bog'lanmagan termoelastiklik masalalar deb ataladi. Odatda, deformatsiyaning birgalikda bo'lish shartlari doirasida tekis termoelastiklik nazaraiyasi masalalari Dyugamel–Neyman munosabatlari yordamida Erining kuchlanish funksiyasi va haroratga nisbatan bigarmonik tenglamani yechishga keltiriladi [6,8]. Bunda T funksiya (harorat maydoni) issiqlik oqimi tenglamasi yechimi sifatida ma'lum deb olinadi. Kuchlanishdagi fazoviy termoelastiklik nazariyasi masalalari Filonenko-Borodich tomonidan koʻrib chiqilgan [3].

Kuchlanishga nisbatan elastiklik nazariyasining chegaraviy masalalari yangi qoʻyilishda quyidagi ishlarda koʻrib chiqilgan [2,4,5].Biz bu ishda temperaturani hisobga olgan holda BeltramiMitchell tenglamasi quyidagi koʻrinishda ifodalanadi:

$$\nabla^2\sigma_{ij}+\frac{1}{1+\nu}S_{,ij}=-\left(X_{i,j}+X_{i,j}\right)-\frac{\nu}{1-\nu}X_{k,k}\delta_{ij}-2\mu\alpha\left(T_{,ij}+\frac{1+\nu}{1-\nu}\delta_{ij}\nabla^2T\right). \qquad (1)$$

Bu yerda σ_{ij}-kuchlanish tenzori, $\nu=\lambda/(\lambda+\mu)/2$ -Puasson koeffitsiyenti, λ,μ-Lame parametrlari, X_i-hajmiy kuchlar, δ_{ij} - Kroneker simvoli, ∇^2 – Laplas operatori.

Agar yuqoridagi (1) tenglamada fazoviy masala sifatida qarasak va hajmiy kuchlar yo`q bo'lsa, u holda quyidagi ifoda keladi:

$$\frac{\partial^2\sigma_x}{\partial x^2}+\frac{\partial^2\sigma_x}{\partial y^2}+\frac{\partial^2\sigma_x}{\partial z^2}=2\mu\alpha\frac{1+\nu}{1-\nu}\left(\frac{\partial^2T}{\partial y^2}+\frac{\partial^2T}{\partial z^2}\right), \qquad (2)$$

$$\frac{\partial^2\sigma_y}{\partial x^2}+\frac{\partial^2\sigma_y}{\partial y^2}+\frac{\partial^2\sigma_y}{\partial z^2}=2\mu\alpha\frac{1+\nu}{1-\nu}\left(\frac{\partial^2T}{\partial x^2}+\frac{\partial^2T}{\partial z^2}\right), \qquad (3)$$

$$\frac{\partial^2\sigma_z}{\partial x^2}+\frac{\partial^2\sigma_z}{\partial y^2}+\frac{\partial^2\sigma_z}{\partial z^2}=2\mu\alpha\frac{1+\nu}{1-\nu}\left(\frac{\partial^2T}{\partial x^2}+\frac{\partial^2T}{\partial y^2}\right), \qquad (4)$$

$$\frac{\partial^2\sigma_{xy}}{\partial x^2}+\frac{\partial^2\sigma_{xy}}{\partial y^2}+\frac{\partial^2\sigma_{xy}}{\partial z^2}=2\mu\alpha\frac{1+\nu}{1-\nu}\frac{\partial^2T}{\partial x\partial y}, \qquad (5)$$

$$\frac{\partial^2\sigma_{xz}}{\partial x^2}+\frac{\partial^2\sigma_{xz}}{\partial y^2}+\frac{\partial^2\sigma_{xz}}{\partial z^2}=2\mu\alpha\frac{1+\nu}{1-\nu}\frac{\partial^2T}{\partial x\partial z}, \qquad (6)$$

$$\frac{\partial^2\sigma_{yz}}{\partial x^2}+\frac{\partial^2\sigma_{yz}}{\partial y^2}+\frac{\partial^2\sigma_{yz}}{\partial z^2}=2\mu\alpha\frac{1+\nu}{1-\nu}\frac{\partial^2T}{\partial y\partial z}. \qquad (7)$$

Bu (2-4)-tenglamalar uch oʻlchovli holatda Beltrami-Mitchell tenglamalarida temperaturani hisobga olingan holdagi tenglamalari.Chegaraviy shartlar esa quyidagicha

$$\sigma_{ij,j}\Big|_{\Sigma}=S_i, \qquad (8)$$

(2-8) tengalamalar termoelastiklik nazariyasi masalasining klassik holatda qo‘yilishini ifodalaydi. Lekin yuqoridagi tenglamalarda chegaraviy shart yetarli bo‘lmaganligi uchun biz prof. Pobedrya tomonidan ishlarida taklif qilingan shartni olamiz. Ya’ni u shart quyidagicha ifodalanadi „Muvozanat tenglamasi soha ichida bajarilsa biz soha chegarasida ham muvozanat tengalamasini ishlatsak bo‘ladi“. Demak biz muvozanat tengalamasini chegaraviy shart sifatida soha chegarasidako‘ramiz:

$$\sigma_{ij,j}\Big|_{\Sigma} = 0, \qquad (9)$$

(2-9) tenglamalar termoelastiklik nazariyasining fazoviy masalasini kuchlanishlarga nisbatan qo‘yilishini ifodalaydi. Bu tenglamalarni chekli ayirmali ko‘rinishga o‘tkazib iteratiya usulida yechamiz.

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