VECTORS AND OPERATIONS ON THEM

Musurmonov Ma'ruf Akram ogli
Teacher of mathematics at Terdu Academic Lyceum

Abstract. This article discusses the concept of vectors, their types, and the basic mathematical operations performed on vectors.
Keywords: Vector, coordinate, modulus, addition, subtraction, scalar product, vector product, unit vector, direction, space.

Vectors are one of the most important concepts in mathematics and physics. They are widely used in engineering, computer science, mechanics, navigation, astronomy, and many other scientific fields. Unlike ordinary numbers, vectors possess both magnitude and direction. This unique property makes vectors essential for describing motion, force, velocity, acceleration, and spatial relationships.
The development of vector theory significantly influenced modern science and technology. Today, vectors are fundamental in linear algebra, geometry, calculus, robotics, machine learning, and computer graphics. Understanding vectors and operations performed on them allows students and researchers to solve complex mathematical and physical problems effectively.
Definition of a Vector
A vector is a quantity that has both magnitude and direction. It is usually represented by a directed line segment or written using boldface letters such as v or arrows such as →v[2].
For example, if a person walks 5 meters east, the movement can be represented as a vector because it includes both distance and direction.
In contrast, scalar quantities have only magnitude. Examples of scalars include mass, temperature, time, and energy.
Representation of Vectors
Vectors may be represented in different forms.
Geometric Representation
Geometrically, a vector is shown as an arrow. The length of the arrow represents the magnitude, while the arrowhead indicates direction.
Coordinate Representation
In coordinate form, vectors are written as ordered pairs or triples.
For example:
v = (3, 4)
This vector means 3 units in the x-direction and 4 units in the y-direction.

In three-dimensional space:
v = (2, -1, 5)
This vector has components along the x, y, and z axes.
Magnitude of a Vector
The magnitude or length of a vector measures its size.
For a two-dimensional vector:
v = (a, b)
the magnitude is:
$|v| = \sqrt{a^2 + b^2}$
For example:
v = (3, 4)
Then:
$|v| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$
In three dimensions:
$|v| = \sqrt{a^2 + b^2 + c^2}$
The magnitude is always nonnegative.
Unit Vectors
A unit vector is a vector whose magnitude equals 1. Unit vectors indicate direction only.
The standard unit vectors are:
i = (1, 0)
j = (0, 1)
k = (0, 0, 1)
Any vector in three-dimensional space can be expressed using these unit vectors.
For example:
v = 2i + 3j - k
Equality of Vectors
Two vectors are equal if they have the same magnitude and direction, regardless of their position in space.
For example:
a = (2, 5)
b = (2, 5)
These vectors are equal because their components are identical.
Types of Vectors
There are several important types of vectors.
Zero Vector
A zero vector has magnitude zero and no specific direction[1].
Negative Vector

A negative vector has the same magnitude as the original vector but opposite direction.

Parallel Vectors

Parallel vectors have the same or opposite directions.

Orthogonal Vectors

Orthogonal vectors are perpendicular to each other.

Position Vector

A position vector describes the location of a point relative to the origin.

Addition of Vectors

Vector addition combines two or more vectors into a single resultant vector.

Graphical Method

Using the triangle rule, the second vector is placed at the end of the first vector. The resultant vector extends from the start of the first vector to the end of the second.

Coordinate Method

If:

$a = (a_1, a_2)$

$b = (b_1, b_2)$

Then:

$a + b = (a_1 + b_1, a_2 + b_2)$

Example:

$(2, 3) + (4, 1) = (6, 4)$

Vector addition is commutative:

$a + b = b + a$

It is also associative:

$(a + b) + c = a + (b + c)$

Subtraction of Vectors

Vector subtraction involves adding the negative vector.

$a - b = a + (-b)$

Example:

$(5, 7) - (2, 3) = (3, 4)$

Graphically, subtraction corresponds to reversing the direction of the vector being subtracted.

Scalar Multiplication

Multiplying a vector by a scalar changes its magnitude.

If:

$v = (a, b)$

Then:

$kv = (ka, kb)$

Example:

3(2, -1) = (6, -3)

If the scalar is positive, the direction remains unchanged. If negative, the direction reverses[1].

Dot Product of Vectors

The dot product, also called the scalar product, combines two vectors and produces a scalar.

For vectors:

$a = (a_1, a_2)$

$b = (b_1, b_2)$

the dot product is:

$a \cdot b = a_1b_1 + a_2b_2$

Example:

(2, 3) · (4, 5) = 8 + 15 = 23

The dot product can also be written as:

$a \cdot b = |a||b|\cos\theta$

where θ is the angle between the vectors.

Applications of the Dot Product

The dot product is used to:

Determine angles between vectors

Check orthogonality

Calculate projections

Solve physics problems involving work and force

Orthogonal Vectors

If:

$a \cdot b = 0$

then the vectors are perpendicular.

Example:

(1, 2) · (2, -1) = 2 - 2 = 0

Therefore, the vectors are orthogonal.

Cross Product of Vectors

The cross product applies only in three-dimensional space.

For vectors:

$a = (a_1, a_2, a_3)$

$b = (b_1, b_2, b_3)$

the cross product is another vector perpendicular to both.

The formula is:

$a \times b = (a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1)$

Applications of the Cross Product

The cross product is used in:
Physics
Engineering
Computer graphics
Rotational mechanics
Electromagnetism
Angle Between Two Vectors
The angle between vectors can be found using the dot product formula:
$\cos\theta = (a \cdot b)/(|a||b|)$
This formula is important in geometry and physics.
Projection of Vectors
The projection of one vector onto another describes the component of one vector in the direction of another.
Projection formulas are widely used in mechanics and engineering.
Linear Combination of Vectors
A vector can be expressed as a combination of other vectors.
For example:
$v = 2a + 3b$
Linear combinations are fundamental in linear algebra.
Vector Equations of Lines
Vectors are used to represent lines in geometry.
The vector equation of a line is:
$r = a + tv$
where:
a is a point on the line
v is the direction vector
t is a scalar parameter
This representation simplifies geometric analysis[3].
Vectors and operations performed on them form one of the most important foundations of mathematics and science. Their ability to represent quantities with both magnitude and direction makes them indispensable in geometry, physics, engineering, computer science, and many other disciplines.
Operations such as addition, subtraction, scalar multiplication, dot products, and cross products allow researchers and students to analyze motion, forces, spatial relationships, and complex systems. Vectors simplify calculations and provide clear visual representations of physical and mathematical concepts.

References

1. Sadullayev A., Xudoyberganov G. Oliy matematika asoslari. – Toshkent: Fan, 2019. – 328 b.
2. Alixonov S. Analitik geometriya va vektorlar algebrai. – Toshkent: O‘qituvchi, 2018. – 256 b.
3. Anton H. Elementary Linear Algebra. – New York: Wiley, 2020. – 624 p.